Two planes on merging routes are:
  --the same distance from the intersection
  --traveling at the same speed.

SMART SKIES

Airspace Systems—Predicting Air Traffic Conflicts

Teacher Guide

Curriculum Supplement 1

Overview of Curriculum Supplement 1

You may choose to spread the experiment and calculation activities over two or three class periods, allowing time for setting up the experiment, conducting the experiment, doing the calculations, and discussing the outcomes.

This is the first in a series of Airspace Systems Curriculum Supplements that address distance-rate-and time problems. The Curriculum Supplement consists of an experiment, worksheets to support the experiment, worksheets for paper-and-pencil calculations, a student analysis of the airspace scenario, and optional pre- and post-assessment instruments.

In this Curriculum Supplement, the controller must merge two flows of air traffic into one stream. Both planes are the same distance from the intersection and both are traveling at the same constant (fixed) speed.

Of the eight *Airspace Systems* Curriculum Supplements, this scenario represents the simplest case.

Airspace Scenario

Students will determine if two airplanes traveling on different merging routes will conflict with one another at the intersection of their flight routes.

The airplanes are the **same distance from the point of intersection**.
The airplanes are traveling at the **same constant (fixed) speeds**.

Objectives

Students will determine the following:

> If two planes are traveling at the same speed on two different routes, and each plane is the same distance from the intersection of the routes, then the planes will meet at the intersection.

Introducing Your Students to the Airspace Scenario

To help your students understand the problem, you can ask them to consider this related problem that is set in a more familiar context:

Two students, Ana and Alex, plan to meet at the movies. Each

student lives the same distance from the theater. Ana and Alex will each leave their homes at the same time and walk at the same constant (fixed) speed.

You can ask your students these questions:

Will Ana and Alex arrive at the movie theater at the same time? Why or why not?
If your students think Ana and Alex will arrive at different times, ask them who will arrive first and why.

Activity 1.0 --

Problem Statement

In a real-world scenario, each plane speed might be 400 nautical miles per hour and each plane might be 40 nautical miles from the point of intersection.

An international nautical mile is 1,852 meters.

A nautical mile per hour is called a "knot".

As a problem extension, you may want to ask your students to solve the problem using real-world data.

Student Handout:
   Worksheet 1.0

Problem Statement
Worksheet 1.0 describes and illustrates the airplane scenario. The speed of each airplane is 1/2 foot/second and each airplane is 20 feet from the point of intersection.

Note: These speeds and distances were chosen to reflect the classroom experiment that the students will conduct and are not related to real-world parameters.

Four questions are posed:

Q1: How many seconds will it take Flight WAL27 to travel 20 feet to the point where the routes come together?

Q2: How many seconds will it take Flight NAL63 to travel 20 feet to the point where the routes come together?

Q3: Will the planes meet at the point where the routes come together?

Q4: If not, how far apart will the planes be when the first plane reaches the point where the routes come together?

Since the planes are traveling at the same constant (fixed) speed and must travel the same distance to the point of intersection, students may be able to answer Question 3 correctly without answering Questions 1 and 2. That is, they may realize that the planes will meet.

Materials
Worksheet 1.0: Problem Statement

Activity 1.1 --

Pretest

Estimated time:
15 - 30 minutes

*The pretest is **optional**.*

Instead of distributing the pretest, you may want to use the questions to guide a classroom discussion.

Student Handout:
   Worksheet 1.1A
   Worksheet 1.1B

Pretest—Make a Prediction
The pretest steps the student through a careful reading of the airplane problem statement. The student is then asked to predict the outcome of the given airplane scenario.

The pretest may be assigned as either an individual or a small-group activity.

If your students have completed other Airspace Systems Curriculum Supplements, you may want to direct them to use a particular calculation method or methods to answer the pretest questions. In that case, Worksheet 1.1B contains blank vertical line plots as well as grids that students can use as they do their calculations.

Materials
Worksheet 1.1A: Pretest—Make A Prediction
Worksheet 1.1B: Lines and Grids

Activity 1.2 --

Experimentation

Estimated time:
   Setup—30 minutes
   Experiment—30 minutes

*For a step-by-step student orientation to the Experiment, see Curriculum Supplement 0, the introduction to **Airspace Systems**.*

Student Handouts:
   Worksheet 1.2A
   Worksheet 1.2B
   Worksheet 1.2C

You may want to give students an overview of the experiment including an explanation of what they will do in each activity.

You may want to ask your

Classroom Experiment
In this small-group activity, students mark off the jet routes on the classroom floor or on an outdoor area. Students assume the roles of pilots, air traffic controllers, and NASA scientists. The pilots step down the jet routes at a prescribed pace. The NASA scientists track and record the pilots' times and the pilots' distances from the intersection of the routes. The air traffic controllers set the pace and measure the separation distance when the first plane arrives at the intersection.

Materials
Activity 1.2A: Set Up the Experiment
        --sidewalk chalk or masking tape
        --measuring tape or ruler
        --marking pens (optional)

Activity 2.2B: Conduct the Experiment
        --1 stopwatch or 1 watch with a sweep second hand or
          1 digital watch that indicates seconds
        --pencils and Data Sheets (Worksheet 1.2C)
        --signs identifying pilots, controllers, and NASA scientists
          Note: the signs are available on the Smart Skies website.

students to compare the experiment distances and speeds with the real-world speeds given in the sidenote for Activity 1.0.

--clipboard (optional)

Student Handouts:
--Worksheet 1.2A: Set Up the Experiment
--Worksheet 1.2B: Conduct the Experiment
--Worksheet 1.2C: Data Sheet

Worksheet 1.2A, Set Up the Experiment

You may want to ask your students to estimate the route layout before they measure.

Students who have little experience in measurement may benefit from first practicing skip counting (by 6) to prepare them to measure 6-inch lengths.

If there is not enough room to set up two 20-foot routes at right angles to one another, another angle may be used. As an alternative, the routes may be set up parallel to each other. (Caution: parallel routes may confuse students who have not had much experience with the experiment. They may not make the connection between the parallel routes and the given merging routes.) In any case, allow enough distance between the routes so that the two pilots are not distracted by one another.

You may want to set up one pair of jet routes as a model that your students can copy.

After a group of students has completed its jet route set-up, you may find it helpful to have them compare their work with another student set-up.

Worksheet 1.2B, Conduct the Experiment

Assign students to positions on 6-8 person teams as follows:

> --Lead Air Traffic Controller (1 student)
> --Secondary Air Traffic Controller (1 student)
> --Pilots (2 students)
> --NASA Scientists, 1 or 2 for each plane (2 – 4 students)

It may be difficult for some student pilots to take 6-inch steps by placing one foot in front of the other. Instead, advise the pilots to place one foot on either side of the jet route and align their toes at each mark. It may be helpful for students to practice.

After the jet routes are set up, have one group of students demonstrate the experiment while the rest of the class observes. Discuss and address any issues that may arise.

Perform the activity at least three times. Compare the results of each trial. Discuss the validity of the results.
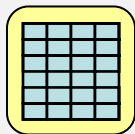
Extensions:

1. Repeat the activity using different students as the Air Traffic Controllers, Pilots, and NASA Scientists.
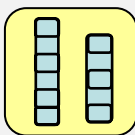
2. Repeat the activity using jet routes longer than 20 feet. Increase the plane speed and the step size to 1 foot/second.

3. Have students draw a scale model of the experiment using real-world data. (See the sidenote for Activity 1.0).

Activity 1.3 --

Calculations

Estimated time:
15 - 30 minutes per worksheet

Calculate the Time for Each Plane to Reach the Intersection
This activity presents six different methods students can use to determine the number of seconds for each plane to arrive at the point where their routes merge.

Each worksheet may be assigned as either an individual or a small-group activity.

You can choose to assign one, some, or all of the worksheets.

You may want to assign some worksheets before and some worksheets after the experiment.

The calculation methods range in order of difficulty as follows:

- □ Counting (completing a table)
- □ Drawing blocks to make a bar graph
- □ Plotting points on two vertical lines
- □ Plotting points on a Cartesian coordinate system
- □ Deriving and using the distance-rate-time formula
- □ Graphing two linear equations

Student Handout:
   Worksheet 1.3A

Worksheet 1.3A, Count Feet and Seconds

Students use patterns and skip-counting to complete a table and solve the problem. At the end of this activity, students may realize it is faster to multiply than to add to obtain the answer.

Prerequisite skills: count by 2s.

Student Handout:
   Worksheet 1.3B

Worksheet 1.3B, Draw Blocks

Students draw blocks, each representing the distance each plane travels in 10 seconds. The students "stack" their blocks along two vertical number lines (one line for each plane) that represent the jet routes.

Notice that the vertical lines are numbered from 20 at the bottom to 0 at the top. Students begin to stack the blocks at the starting point of each plane, 20 feet away from the intersection. The

intersection of the routes is represented with 0 at the top of each number line.

To help students make the connection between the "inverted Y" jet routes and the vertical scales, students are first asked to plot a point on the original jet route diagram and then stack the corresponding block along the vertical scale.

Prerequisite skills: read and build a bar graph with a vertical scale marked in 1-foot units; count by 10s.

Worksheet 1.3C, Plot Points on Two Vertical Lines

Student Handout:
Worksheet 1.3C

This graph is similar to the way families record and compare the height of their children at the same ages. They mark off each child's birthday height (distance from the floor) on a doorway and then record their age (time since birth) beside the height mark.

The students plot their points along two vertical number lines (one line for each plane) that represent the jet routes.

Notice that the vertical lines are numbered from 20 at the bottom to 0 at the top. The bottom of each number line represents the starting point of each plane, 20 feet away from the intersection. The intersection of the routes is represented with 0 at the top of each number line.

To help students make the connection between the "inverted Y" jet routes and the vertical scales, students are first asked to plot a point on the original jet route diagram and then plot the corresponding point on the vertical scale.

Prerequisite skills: plot a point on a (vertical) number line.

Worksheet 1.3D, Plot Points on a Cartesian Coordinate System

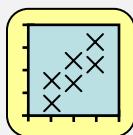Student Handout:
Worksheet 1.3D

Notice that the vertical axis is numbered from 0 at the top to **negative** 20 at the bottom. The numbers along the vertical axis represent the distance (with a negative sign attached) from the point where the two routes meet. Negative numbers are used because the points lie below the horizontal axis (the horizontal line at 0 feet).

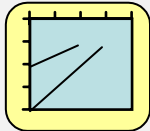Prerequisite skills: plot a point on a Cartesian coordinate system (the xy-plane)
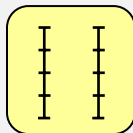
Student Handout:
Worksheet 1.3E

d = r • t

Student Handout:
Worksheet 1.3F

t = d / r

Student Handout:
Worksheet 1.3G

Caution: Students may confuse the path of a plane with the graph of the plane's distance from the intersection as a function of time. In particular, students need to understand that the routes meet, but the planes do not necessarily meet.

Extension (optional):
For each plane, connect the points with a straight line. Find the equation of each line.

Worksheet 1.3E, Derive the Distance-Rate-Time Formula

Students use patterns to derive the distance-rate-time formula in the form d = rt.

Prerequisite skills:
Use patterns to make a generalization.

Worksheet 1.3F, Use the Distance-Rate-Time Formula

Students apply the distance-rate-time formula in the form t = d/r.

Prerequisite skills:
Substitute numbers into a formula.

Worksheet 1.3G, Graph Two Linear Equations

Notice that the points are plotted in the fourth quadrant. So the given portion of the y- axis is numbered from 0 at the top to **negative** 20 at the bottom. The numbers along the y-axis represent the distance (with a negative sign attached) from the point where the two routes meet. Negative numbers are used because the points lie below the x-axis.

Prerequisite skills:
Graph a linear equation by making a table of ordered pairs.
Find the slope of a line given the equation of the line and the graph of the line.

Extension (optional):
You may want to ask your students to find the intercepts of each line and interpret those intercepts in the context of the airspace problem.

For each plane, the y-intercept represents the plane's initial distance (with negative sign attached) from the intersection point.

For each plane, the x-intercept represents the number of seconds for the plane to reach the intersection point.

The horizontal distance between the x-intercepts represents the

number of seconds between the arrival of the two planes at the intersection.

Materials
Worksheet 1.3A: Calculate the time—count feet & seconds
Worksheet 1.3B: Calculate the time—draw blocks
Worksheet 1.3C: Calculate the time—plot on two vertical
                                          scales
Worksheet 1.3D: Calculate the time—plot points on a Cartesian
                                          coordinate system
Worksheet 1.3E: Derive the Distance-rate-time formula
Worksheet 1.3F: Use the Distance-rate-time formula
Worksheet 1.3G: Graph Two Linear Equations

Activity 1.4 --

Analysis

Estimated time:
45 minutes

Student Handout:
   Worksheet 1.4

Compare the Experimental Results with the Predicted Results
Students compare the outcome of the experiment with their pretest predictions.

This activity may be assigned as either an individual or a small-group activity.

If you assigned some calculation worksheets (Activity 1.3) prior to the experiment, students can compare their calculations with the experimental results.

You may want to assign some Activity 1.3 calculation worksheets after the experiment to give students another basis for comparison.

As part of the Analysis, you may also want to ask your students to create a similar problem in a different setting. They have already considered a problem in which two students walk from their respective homes to a movie theater. (See the Airspace Scenario section of this document.)

Now, you might suggest they consider two cars traveling in parallel lanes on the same road, with the two lanes merging into one lane. Each car is the same distance from the merge and the cars are traveling at the same constant (fixed) speed. Students should realize that the cars will arrive at the merge at the same time.

Note: To be consistent with the airspace scenarios, it is important that for each problem created by you or your students, you choose a fixed (constant) speed for each vehicle or person. (For example, a rocket launch scenario would *not* be appropriate because a launched rocket typically accelerates and therefore its speed is not

constant.)

Materials
Worksheet 1.4: After the Experiment

Activity 1.5 --

Posttest

Estimated time:
15 - 30 minutes

*The posttest is **optional**.*

Student Handouts:
  Worksheet 1.5
  Worksheet 1.1B

Curriculum Supplement Posttest

This activity may be assigned as either an individual or a small-group activity.

You can direct your students to use a particular calculation method or methods to answer the posttest questions. Worksheet 1.1B (used for the Pretest) contains blank vertical line plots and grids that students can use as they do their calculations.

Materials
Worksheet 1.5: Posttest
Worksheet 1.1B: Lines and Grids

Problem Statement

In the picture below, two airplanes are flying on different routes.

The World Airlines plane has flight number **WAL**27.
The speed of Flight WAL27 is 1/2 foot/second (0.15 meters/second).

The National Airlines plane has flight number **NAL**63.
The speed of Flight NAL63 is 1/2 foot/second (0.15 meters/second).

Flight WAL27 is 20 feet (6.1 meters) away from the point where the two routes come together.
Flight NAL63 is 20 feet (6.1 meters) away from the point where the two routes come together.

20 feet   20 feet

WAL27   NAL63
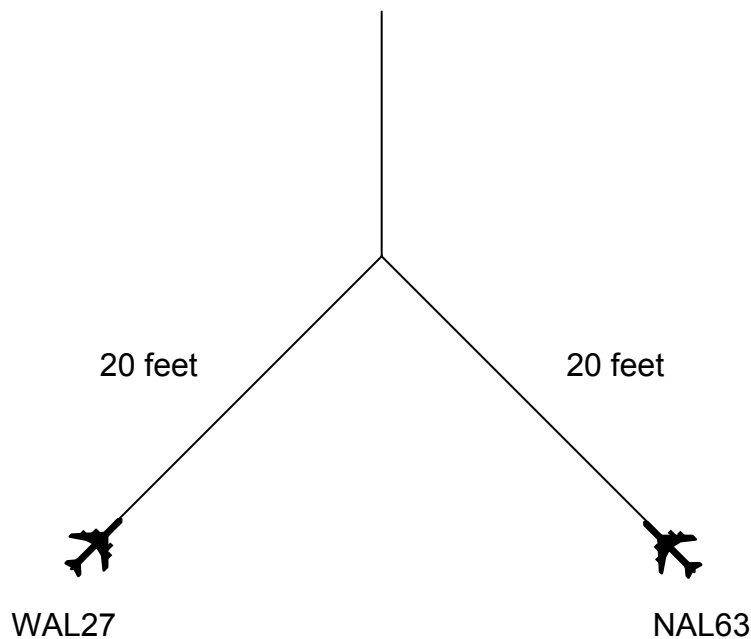
Question 1: How many seconds will it take Flight WAL27 to travel 20 feet to the point where the two routes come together?

Question 2: How many seconds will it take Flight NAL63 to travel 20 feet to the point where the two routes come together?

Question 3: Will the planes meet at the point where the two routes come together?

Question 4: If not, how many feet apart will the planes be when the first plane reaches the point where the routes come together?

Pretest—Make a Prediction

In the picture below, two airplanes are flying on different routes.

1. Draw a circle around the point where the routes come together.

The World Airlines plane has flight number WAL27.
The speed of Flight WAL27 is 1/2 foot/second.

2. Write the speed of Flight WAL27 next to its picture.

3. How far does Flight WAL27 travel in one second?   _____

 The National Airlines plane has flight number NAL63.
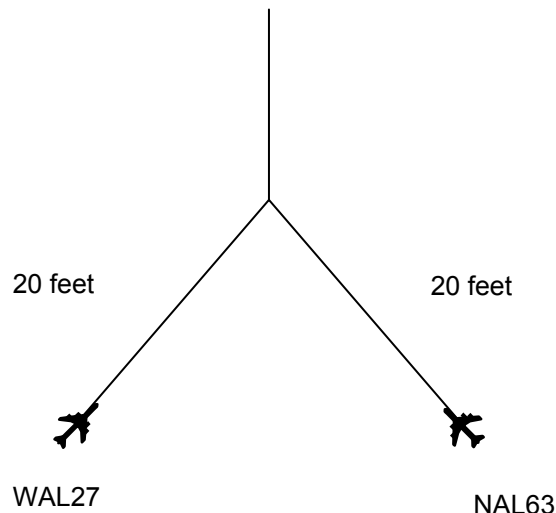 The speed of Flight NAL63 is 1/2 foot/second.

4. Write the speed of Flight NAL63 next to its picture.

5. How far does Flight NAL63 travel in one second?   _____

6. Do you think that the two planes will meet at the point
   where the two routes come together?   _____

   Why or why not?   _____
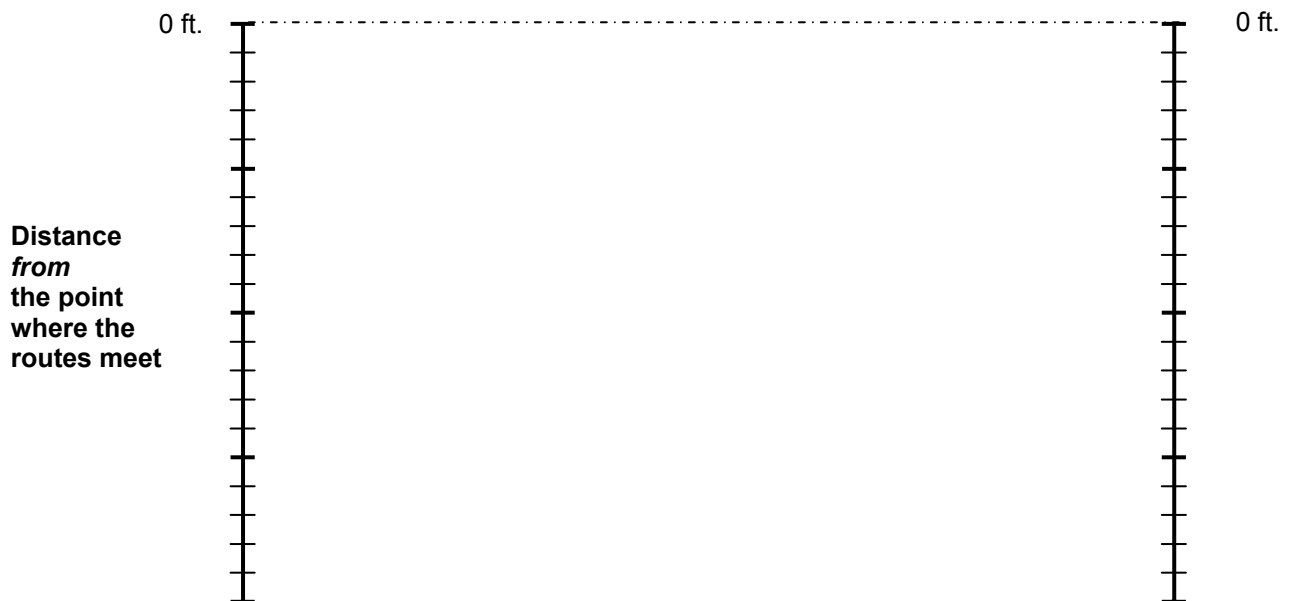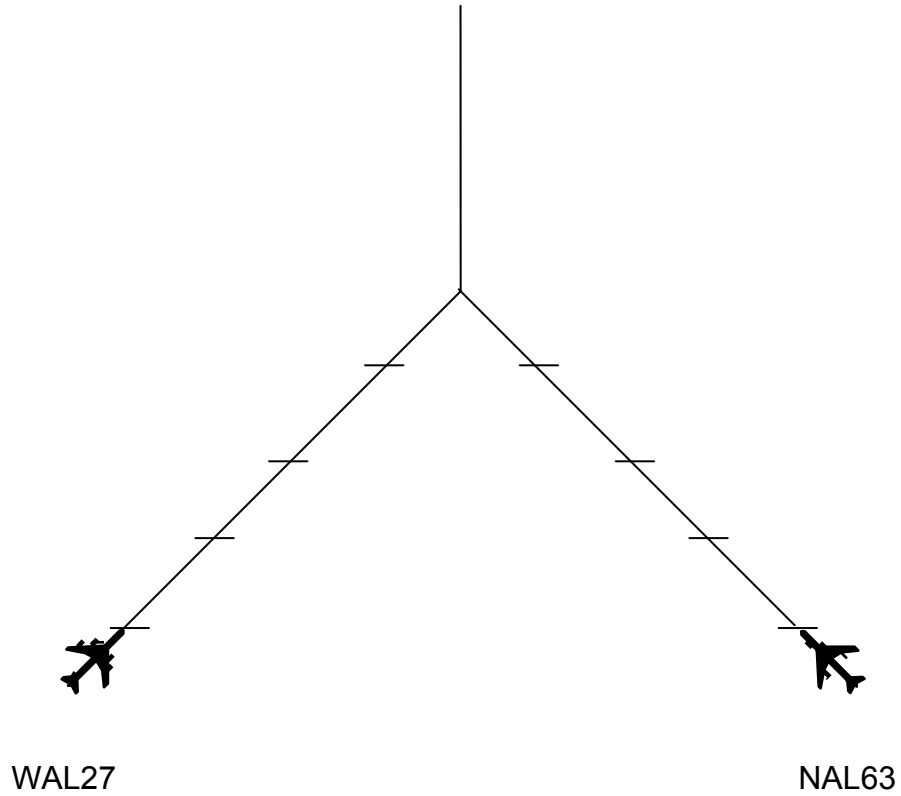   _____

7. If not, how many feet apart do you think the planes will be when the first plane
   reaches the point where the routes come together?   _____

20 feet   20 feet

WAL27   NAL63

Lines and Grids

WAL27   NAL63

0 ft.   0 ft.

Distance
from
the point
where the
routes meet

Lines and Grids

WAL27   NAL63

Time traveled

0 sec.

0 ft.

**Distance
(with negative
sign attached)
from the point
where the two
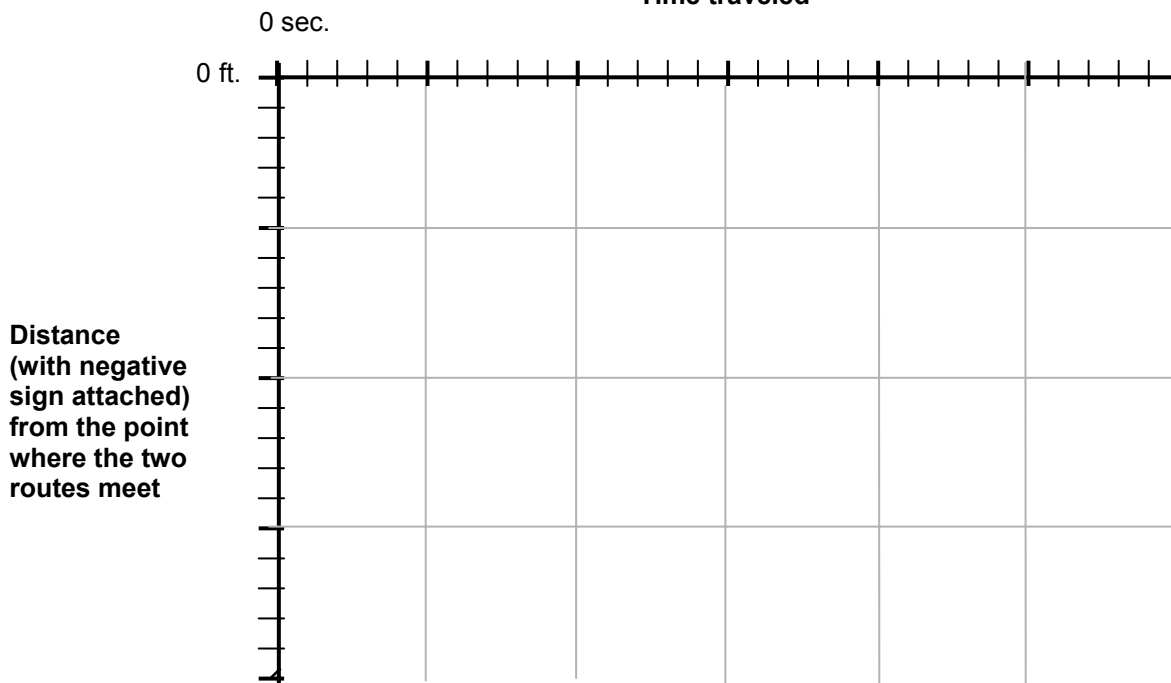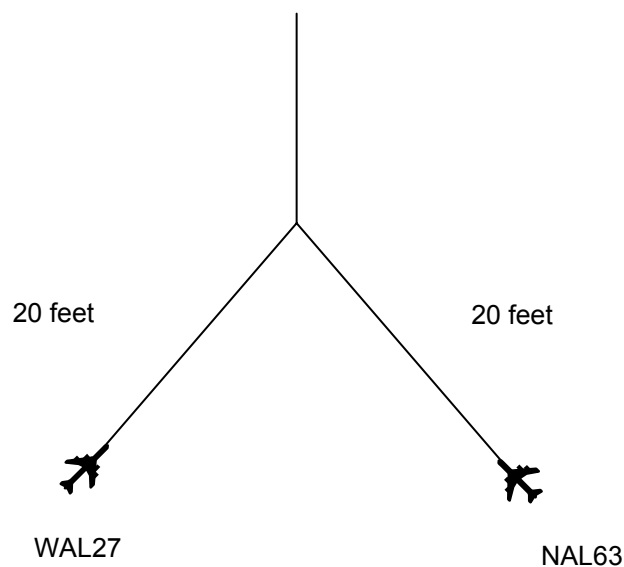routes meet**
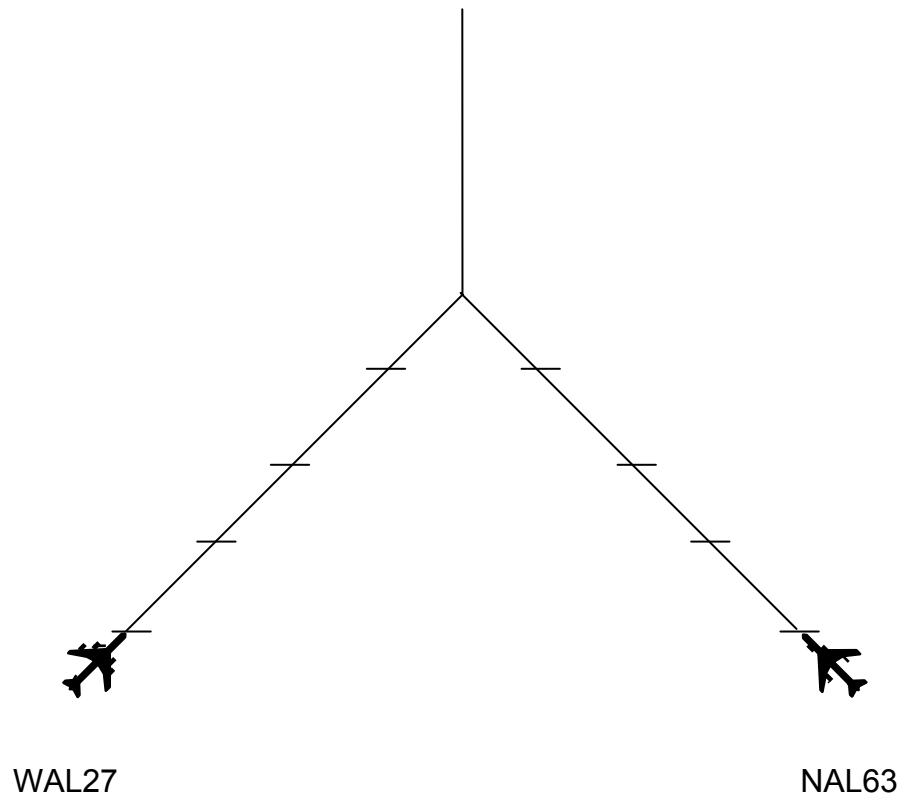
Set Up the Experiment

1. Use sidewalk chalk (or masking tape) to lay out 2 jet routes, one for each
    airplane as shown below.
    Each route should be 20 feet long.

2. The speed of Flight WAL27 is 1/2 foot/second. Stand at the beginning of the
    jet route for Flight WAL27.   Place a mark (or a piece of masking tape) every
    1/2 foot (every 6 inches) along the jet route. This will guide the pilot as he or
    she steps down the jet route.

3. The speed of Flight NAL63 is 1/2 foot/second. Stand at the beginning of the
    jet route for Flight NAL63.   Place a mark (or a piece of masking tape) every
    1/2 foot (every 6 inches) along the jet route. This will guide the pilot as he or
    she steps down the jet route.

4. On each jet route, place and label a longer chalk mark (or longer piece of
    masking tape) at the following positions:
    5 feet from the start, **10 feet** from the start, **15 feet** from the start, the **finish point**
    Note: The finish point is where the jet routes meet.

20 feet   20 feet

WAL27   NAL63

_____

Name

Conduct the Experiment

1. **Review your prediction.**

   Do you think the airplanes will meet at the point where the two routes meet?
   Why or why not?

2. **Take your position.** Circle your role in the diagram and in the following list:

   Lead Air Traffic Controller: Give the command "Take your ready positions."
   Pilots: Position yourself at the start of your jet route.
   Secondary Controller: Take your data sheet, measuring tape, and pencil and go to your controller location as shown below.
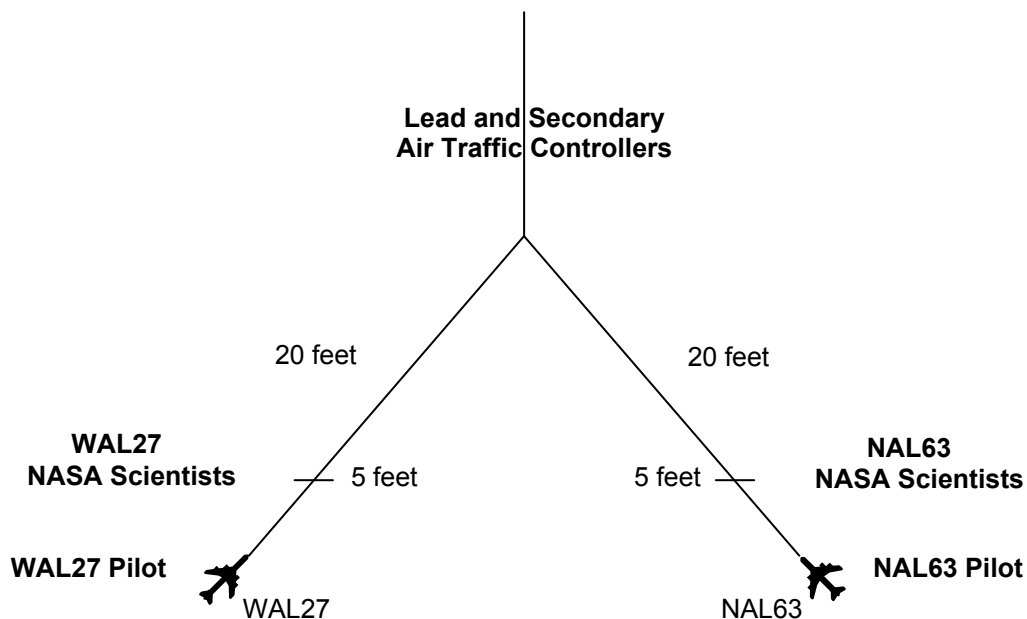   **NASA Scientists:** Take your data sheet and pencil and go to your first observation position at the 5-foot line as shown below.

3. **Get ready to begin.** Circle your role in the following list:

   Lead Air Traffic Controller: Give the command "Set."
   Pilots: Prepare to step down your jet route. You may want to practice first. It helps to keep one foot on each side of the jet route.
   NASA Scientists: Get ready to measure and record the information on the data sheet.

**Lead and Secondary
Air Traffic Controllers**

20 feet   20 feet

WAL27   **NAL63**
NASA Scientists   5 feet   5 feet   **NASA Scientists**

WAL27 Pilot   **NAL63 Pilot**
           WAL27   NAL63

4. **Begin the experiment.** Circle your role in the following list:

Lead Air Traffic Controller: Give the command "Ready." Start your stopwatch and count the seconds aloud, "One, two, three…" and so on.

Pilots: Take your first step on count "One." Each second, take one step to the next timing mark.

NASA Scientists: Record the time your aircraft arrives at the 5-foot line. Stay ahead of the pilot and record the time your aircraft arrives at the 10-foot line, the 15-foot line, and the point where the Controller says, "Halt."

5. **End the experiment.** Circle your role in the following list:

Secondary Controller: When the first Pilot reaches the point where the two routes meet, give the command "Halt." Measure and record the separation distance between the planes. To do this, measure the distance of the second Pilot from the point where the two routes meet.

Lead Air Traffic Controller: Stop counting the seconds when you hear "Halt."

Pilots: Stop and remain where you are on the jet route when you hear "Halt."

NASA Scientists: Record the "Halt" time.

Data Sheet

a. Fill in this table:

Flight Number	Speed	Distance from the Point Where the Two Routes Meet
WAL27		
NAL63		

b. On the picture below, circle your job title. Notice the data you need to record.

c. During Experiments 1, 2, and 3, record your data.

1	2	3

Separation Distance (feet) at Intersection

1	2	3

Air Traffic Controllers

20'———— 20'

"HALT" Time

1	2	3

Time (seconds)

1	2	3

←————— 15'   15'————→

Time (seconds)

1	2	3

←————— 10'   10'————→

Time (seconds)

1	2	3

←——— 5'   5' ———→

NASA Scientist: WAL27
Start recording here.

"HALT" Time

1	2	3

Time (seconds)

1	2	3

Time (seconds)

1	2	3

Time (seconds)

1	2	3

NASA Scientist: NAL63
Start recording here.

START———— 0'   0'———— START
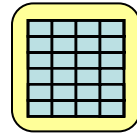
Pilot: WAL27   **Pilot: NAL63**
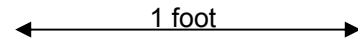
PILOT

AIR TRAFFIC CONTROLLER

NASA SCIENTIST

How Much Time To Reach the Point Where the Two Routes Meet?

(Count Feet and Seconds to Find the Answer)

The speed of each airplane is 1/2 foot per second.

That means each airplane travels 1/2 foot in 1 second.

1/2 foot

So each airplane travels 1 foot in 2 seconds.

1 foot

Each airplane starts 20 feet from the point where the two routes meet.

1. Fill in the given table to see how many seconds it will take each plane to travel 20 feet.

After you fill in the table, answer the following questions:

2. How many seconds will it take each plane to travel 20 feet and arrive at the point where the two routes meet?

    WAL27 _____ seconds   NAL63 _____ seconds

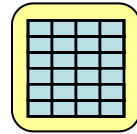3. Will the two planes meet at the point where the two routes come together?   _____

4. Why or why not? _____

    _____

5. You filled in the table to find the answer. Can you think of a faster way to find the answer? If so, describe the faster way. _____

    _____

6. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? _____
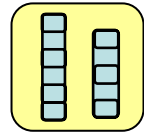
    _____

Flight WAL27		Flight NAL63	
How many feet?	How many seconds?	How many feet?	How many seconds?
1	2	1	2
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

How Much Time To Reach the Point Where the Two Routes Meet?
(Draw Blocks to Find the Answer)

Each airplane starts 20 feet from the point where the two routes meet.

The speed of each airplane is 1/2 foot per second.

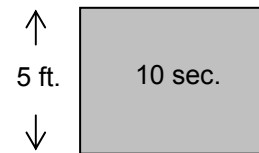In 1 second, each airplane travels 1/2 foot.

In 2 seconds, each airplane travels 1 foot.

In 10 seconds (5 x 2 seconds), each plane travels 5 x 1 foot.

That is, each plane travels 5 feet in 10 seconds.

The height of this block represents 5 feet, the

distance each plane travels in 10 seconds.

↑
5 ft.   10 sec.
↓

Now you will use blocks and dots to plot the position of each plane as it travels to where the two routes meet.

Look at the top picture on page 4. The picture shows the two jet routes.

Flight NAL63 started 20 feet away from the point where the two routes meet.

After 10 seconds, Flight NAL63 has moved 5 feet closer to that point.

So the plane is 15 feet from the point where the two routes meet.

A dot shows the position of Flight NAL63 after 10 seconds.

Flight WAL27 started 20 feet away from the point where the two routes meet.

After 10 seconds, Flight WAL27 has moved 5 feet closer to that point.
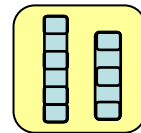
So the plane is 15 feet from the point where the two routes meet.

A dot shows the position of Flight WAL27 after 10 seconds.

The dots are connected with a line marked "10 seconds."

Next look at the bottom picture on page 4. The picture shows a bar graph made of blocks.

A block shows the position of Flight NAL63 after 10 seconds.
A block shows the position of Flight WAL27 after 10 seconds.

The blocks are connected with a line marked "10 seconds."

Now it's your turn to draw and connect.

- ☐ On the top picture on page 4, draw a dot to show the position of Flight NAL63 after 20 seconds.
- ☐ On the bottom picture on page 4, trace the block that shows the position of Flight NAL63 after 20 seconds.

- ☐ On the top picture, draw a dot to show the position of Flight WAL27 after 20 seconds.
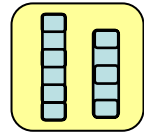- ☐ On the bottom picture, trace the block that shows the position of Flight WAL27 after 20 seconds.

- ☐ Connect your dots with a line marked "20 seconds."
- ☐ Connect your blocks with a line marked "20 seconds."

- ☐ Draw and connect dots at 30 seconds.
- ☐ Draw and connect blocks at 30 seconds.

- ☐ Keep going until the first plane reaches the place where the two routes meet.

When you are done, answer these questions.

1. How many seconds will it take each plane to travel 20 feet and arrive at the point where the two routes meet?
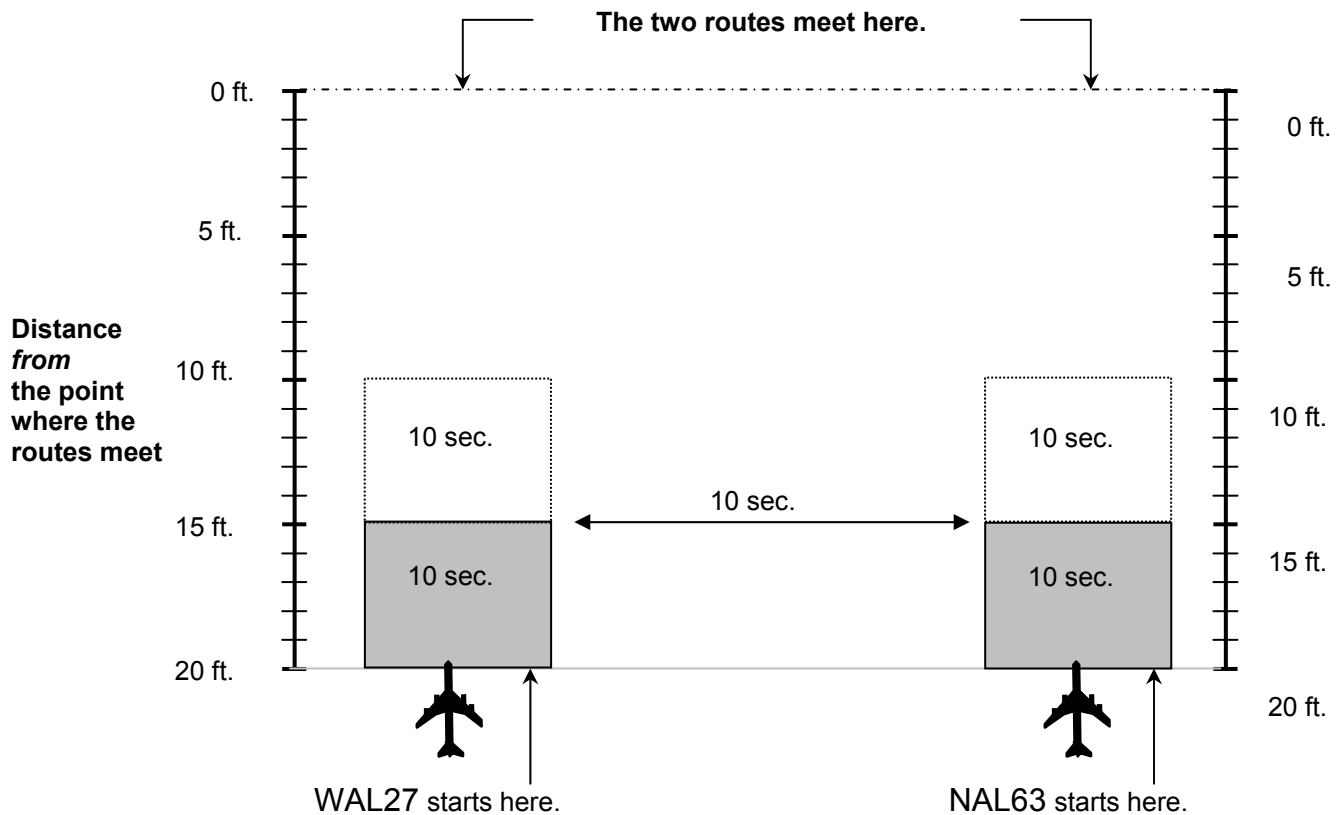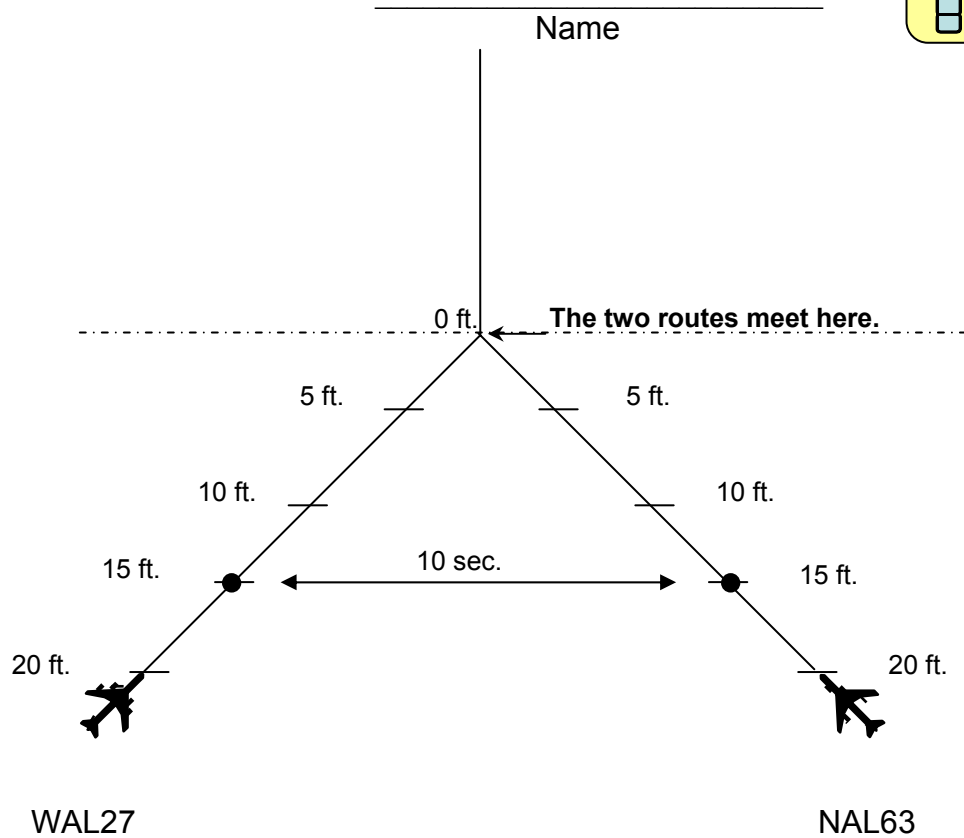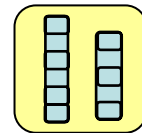
    WAL27 _____ seconds   NAL63 _____ seconds

2. Will the two planes meet at the point where
    the two routes come together?   _____

3. Why or why not? _____

4. If you think the two planes will meet, what could you tell the air traffic
    controllers to do to avoid a collision? _____

_____
Name

0 ft. The two routes meet here.

5 ft.   5 ft.

10 ft.   10 ft.

15 ft.   10 sec.   15 ft.

20 ft.   20 ft.

WAL27   NAL63

The two routes meet here.

Distance _from_ the point where the routes meet

0 ft.   0 ft.

5 ft.   5 ft.

10 ft.   10 ft.

10 sec.   10 sec.   10 sec.

15 ft.   15 ft.

10 sec.   10 sec.

20 ft.   20 ft.

WAL27 starts here.   NAL63 starts here.
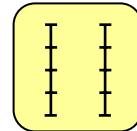
How Much Time To Reach the Point Where the Two Routes Meet?
(Plot Points on Lines to Find the Answer)

Each airplane is 20 feet from the point where the two routes meet.

The speed of each airplane is 1/2 foot per second.

In 1 second, each airplane travels 1/2 foot.
In 2 seconds, each airplane travels 1 foot.

In 10 seconds (5 x 2 seconds), each plane travels 5 x 1 foot.
That is, each plane travels 5 feet in 10 seconds.

You will use an **O** to show the position of Flight NAL63.
You will use an **X** to show the position of Flight WAL27.

Look at the top picture on page 4. The picture shows the two jet routes.

Flight NAL63 started 20 feet away from the point where the two routes meet.
After 10 seconds, Flight NAL63 has moved 5 feet closer to that point.
So the plane is 15 feet from the point where the two routes meet.
An **O** shows the position of Flight NAL63 after 10 seconds.

Flight WAL27 started 20 feet away from the point where the two routes meet.
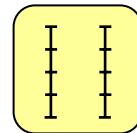After 10 seconds, Flight WAL27 has moved 5 feet closer to that point.
So the plane is 15 feet from the point where the two routes meet.
An **X** shows the position of Flight WAL27 after 10 seconds.

The **X** and **O** are connected with a line marked "10 seconds."

Next look at the bottom picture on page 4. The picture shows two vertical line graphs.

An **O** shows the position of Flight NAL63 after 10 seconds.

An **X** shows the position of Flight WAL27 after 10 seconds.

The **X** and **O** are connected with a line marked "10 seconds."

Now it's your turn to draw and connect.

- □ On the top picture on page 4, draw an **O** to show the position of Flight NAL63 after 20 seconds.
- □ On the bottom picture on page 4, draw an **O** to show the position of Flight NAL63 after 20 seconds.

- □ On the top picture, draw an **X** to show the position of Flight WAL27 after 20 seconds.
- □ On the bottom picture, draw an **X** to show the position of Flight WAL27 after 20 seconds.
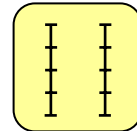
- □ On the top picture, connect the **X** and **O** with a line marked "20 seconds."
- □ On the bottom picture, do the same thing.

- □ On the top picture, draw and connect an **X** and an **O** at 30 seconds.
- □ On the bottom picture, do the same thing.

- □ Keep going until the first plane reaches the place where the two routes meet.

When you are done, answer these questions.

1. How many seconds will it take each plane to travel 20 feet and arrive at the point where the two routes meet?

   WAL27 _____ seconds   NAL63 _____ seconds

2. Will the two planes meet at the point where
   the two routes come together?   _____

3. Why or why not? _____

4. If you think the two planes will meet, what could you tell the air traffic
controllers to do to avoid a collision? _____

Name _____

0 ft. The two routes meet here.

5 ft.   5 ft.

10 ft.   10 ft.

15 ft.   ✕   ←—— 10 sec. ——→   ⊖   15 ft.

20 ft.   ←——— 0 sec. ———→   20 ft.

WAL27   NAL63

The two routes meet here.

0 ft.   0 ft.

5 ft.   5 ft.

Distance _from_ the point where the routes meet

10 ft.   10 ft.

15 ft.   ✕ ←———— 10 sec. ————→ ⊕   15 ft.

20 ft.   ←———— 0 sec. ————→   20 ft.

WAL27 starts here.   NAL63 starts here.

How Much Time To Reach the Point Where the Routes Meet?
(Plot Points on a Grid to Find the Answer)

The speed of each airplane is 1/2 foot per second.

In 2 seconds, each airplane travels 1 foot.

In 10 seconds, each plane travels 5 feet.

Each plane starts 20 feet from the point where the two routes meet.

After 10 seconds, each plane has traveled 5 feet.

So after **10** seconds, each plane is **15** feet *from* the point where the two routes meet.

On each jet route on page 3, we represent that information with a mark (**X** or **O**) at 15.

The marks are connected with a line marked "10 seconds."

On the grid on page 3, we represent that information with the point (**10**, **-15**).

We use **negative** fifteen because the point lies **below** the horizontal line at 0 feet where the two routes meet.

The **O** at the point (10, -15) shows the position of Flight NAL63 after 10 seconds.

The **X** at the point (10, -15) shows the position of Flight WAL27 after 10 seconds.

Now it's your turn to plot and connect points on the routes and to plot points on the grid on page 3.

Put an **O** at the point that shows the position of Flight NAL63 after 20 seconds, 30 seconds and so on.

Put an **X** at the point that shows the position of Flight WAL27 after 20 seconds, 30 seconds, and so on.
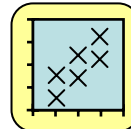
Keep going until the first plane reaches the horizontal line at 0 ft.

That line represents the point where the two routes meet.

When you have finished plotting points, answer these questions.

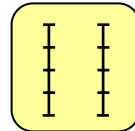1. How many seconds will it take each plane to travel 20 feet and arrive at the point where the two routes meet?

     WAL27 _____ seconds   NAL63 _____ seconds

2. Will the two planes meet at the point where
    the two routes come together?   _____

3. Why or why not? _____

4. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? _____

Horizontal line at 0 feet

0 ft.

5 ft.   5 ft.

10 ft.   10 ft.

10 sec.

15 ft.   15 ft.

20 ft.   0 sec.   20 ft.

WAL27   NAL63

Time traveled

0 secs.	10 secs.	20 secs.	30 secs.	40 secs.	50 secs.	60 secs.

0 ft.   ← **Horizontal line at 0 feet**

Distance (with negative sign attached) *from* the point where the routes meet

- 5 ft.

- 10 ft.

- 15 ft.

- 20 ft.

X   WAL27

O   NAL63
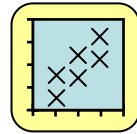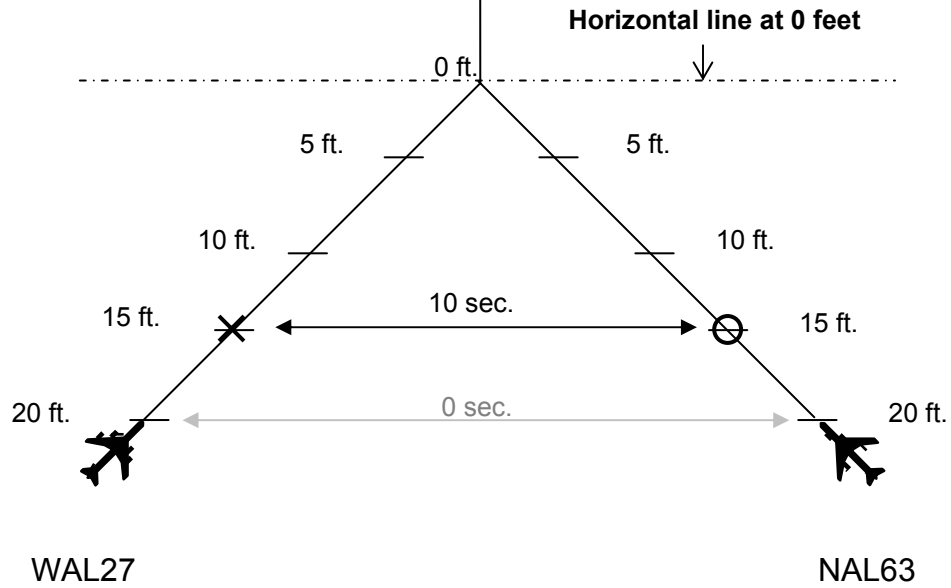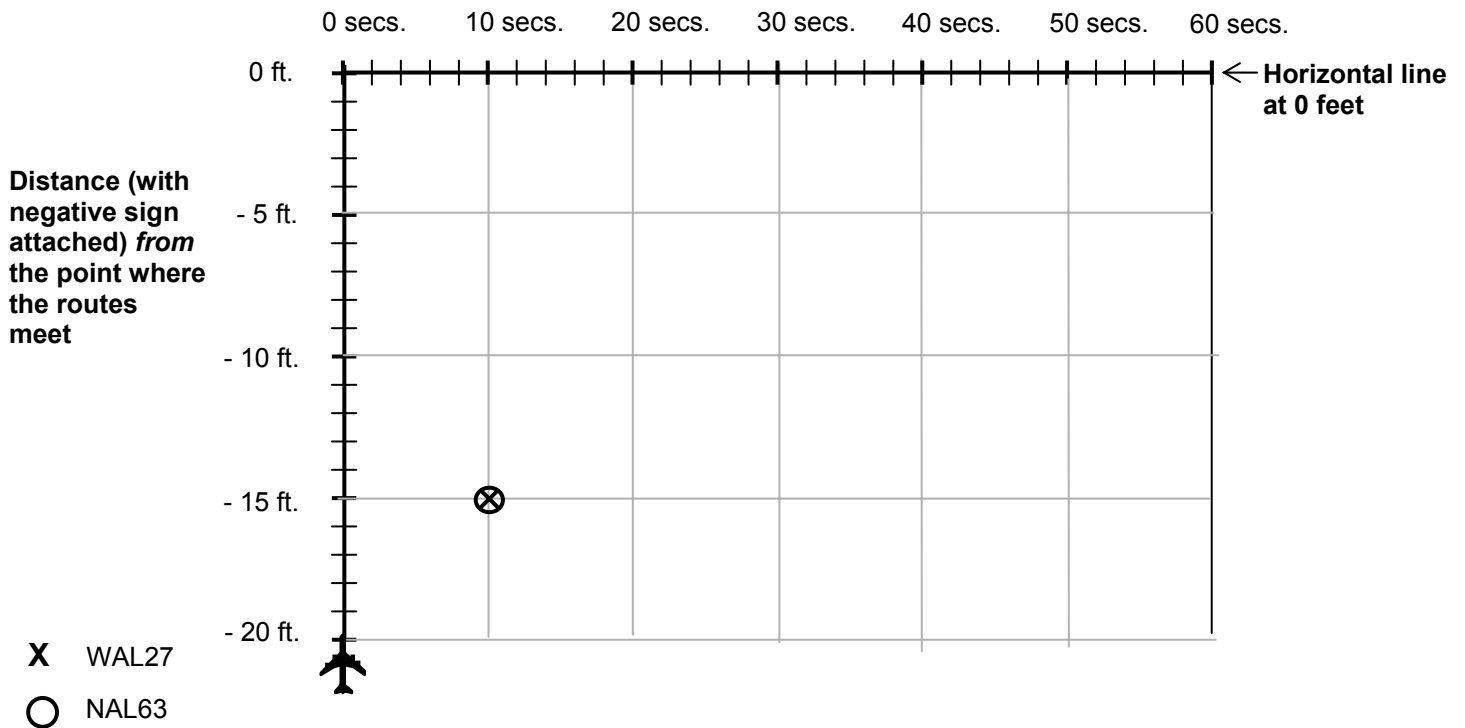
$$d = r \cdot t$$

Derive the Distance-Rate-Time Formula

Each airplane starts 20 feet from the point where the two routes meet.

The speed of each airplane is 0.5 feet per second.

In 1 second, each plane travels 0.5 feet.

In 2 seconds, each plane travels 0.5 feet/second $\times$ 2 seconds = 1.0 foot.

In 3 seconds, each plane travels 0.5 feet/second $\times$ 3 seconds = 1.5 feet.

1. In 4 seconds, each plane travels _____ $\times$ _____ = _____ feet.

2. In 5 seconds, each plane travels _____ $\times$ _____ = _____ feet.

3. In 6 seconds, each plane travels _____ $\times$ _____ = _____ feet.

4. How could you use multiplication to find the distance each plane travels in 14 seconds?

_____

_____

One way to find the distance is to multiply the plane's speed by 14 seconds, like this:

$$0.5 \text{ feet/second} \times 14 \text{ seconds} = 7 \text{ feet}$$

This suggests the following rule:

To find the distance traveled, multiply the speed by the time traveled.

In mathematics and science, we often say "rate" instead of "speed."

Then we can write the rule like this:

distance = rate $\times$ **time**

This relationship is called the Distance-Rate-Time Formula. We often write it like this:

$$d = r \cdot t$$

5. Use the formula to find the distance traveled by each plane in 20 seconds.

In 20 seconds, each plane travels _____ feet.

_____
Name

Use the Distance-Rate-Time Formula

Each airplane starts 20 feet from the point where the two routes meet.

The speed of each airplane is 0.5 feet per second.

In 1 second, each plane travels 0.5 feet.

In 2 seconds, each plane travels 0.5 feet/second × 2 seconds = 1.0 foot.

In 3 seconds, each plane travels 0.5 feet/second × 3 seconds = 1.5 feet.

To find the distance traveled after t seconds, we multiply the rate by the time:

distance traveled = **r**ate of travel × **t**ime traveled

This relationship is called the Distance-Rate-Time Formula. We often write it like this:

$$d = r \cdot t$$

If we divide both sides of this equation by r, we get a formula for time traveled:

$$t = \frac{d}{r}$$

$$\frac{d}{r} = \frac{r \cdot t}{r}$$

You can use this formula to find the number of seconds for Flight WAL27 to travel 20 feet to the point where the two routes meet.

Flight WAL27 $\quad t = \dfrac{20 \text{ feet}}{0.5 \text{ feet per second}}$ = _____ seconds

(Hint: Divide 20 by 0.5 .)

Use the same formula to find the number of seconds for Flight NAL63 to travel 20 feet to the point where the two routes meet.

Flight NAL63 _____ seconds

_____
Name

Now answer these questions.

1. How many seconds will it take each plane to travel 20 feet and arrive at the point
   where the two routes meet?
       WAL27 _____ seconds   NAL63 _____ seconds

2. Will the two planes meet at the point where
   the two routes come together?   _____

3. Why or why not? _____

4. If you think the two planes will meet, what could you tell the air traffic
   controllers to do to avoid a collision? _____

20 feet   20 feet

WAL27   NAL63
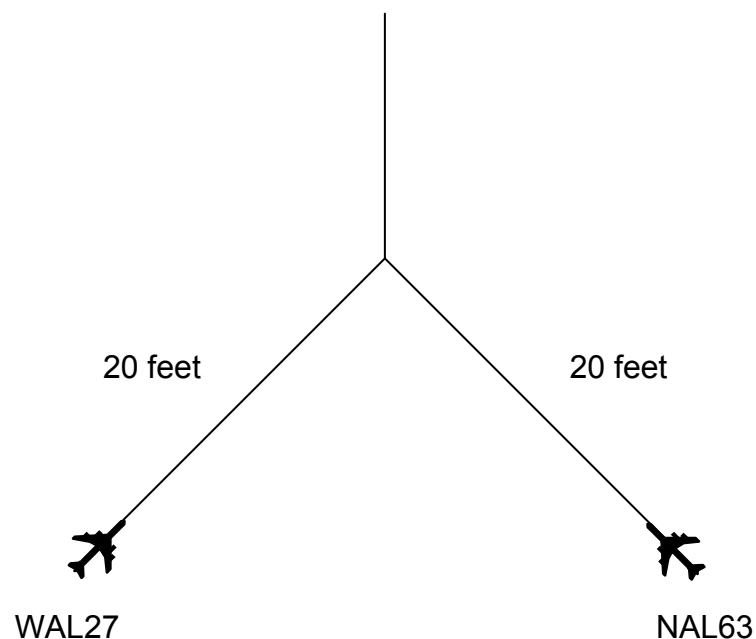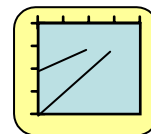
How Much Time To Reach the Point Where the Two Routes Meet?

(Graph Two Linear Equations to Find the Answer)

We can use a linear equation to describe the position of an airplane that travels at a constant speed.

We begin with Flight WAL27.

The speed of Flight WAL27 is 0.5 feet per second.

When the clock starts at 0 seconds, the plane is 20 feet from the point where the two routes meet.

The position of Flight WAL27 is given by this equation:

y = 0.5x – 20   **WAL27**

Here:
       x = the time traveled (in seconds) and
       y = the number of feet (with a negative sign attached)
          from the intersection of the two routes.

Notice that when x = 0, y = –20.

This means that when the clock starts at 0 seconds, the plane is 20 feet from the point where the two routes meet.

Now think about Flight NAL63.

Since the speed of Flight NAL63 is also 0.5 feet per second and its distance from the intersection is also 20 feet, the position of Flight NAL63 is also given by this equation:

y = 0.5x – 20   **NAL63**
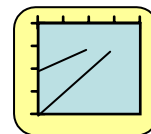
Complete each table and use the ordered pairs to graph each line.

WAL27   y = 0.5x – 20   NAL63   y = 0.5x – 20

x	y
0	
10	
20	
30	

x	y
0	
10	
20	
30	

Use a solid line for the graph of **Flight WAL27**. _____

Use a dotted line for the graph of **Flight NAL63**. ·······························

Time traveled

0 secs.   10 secs.   20 secs.   30 secs.   40 secs.   50 secs.   60 secs.

0 ft.   **x-axis**

Distance (with negative sign attached) from the point where the routes meet

- 5 ft.

- 10 ft.

- 15 ft.

- 20 ft.

y-axis
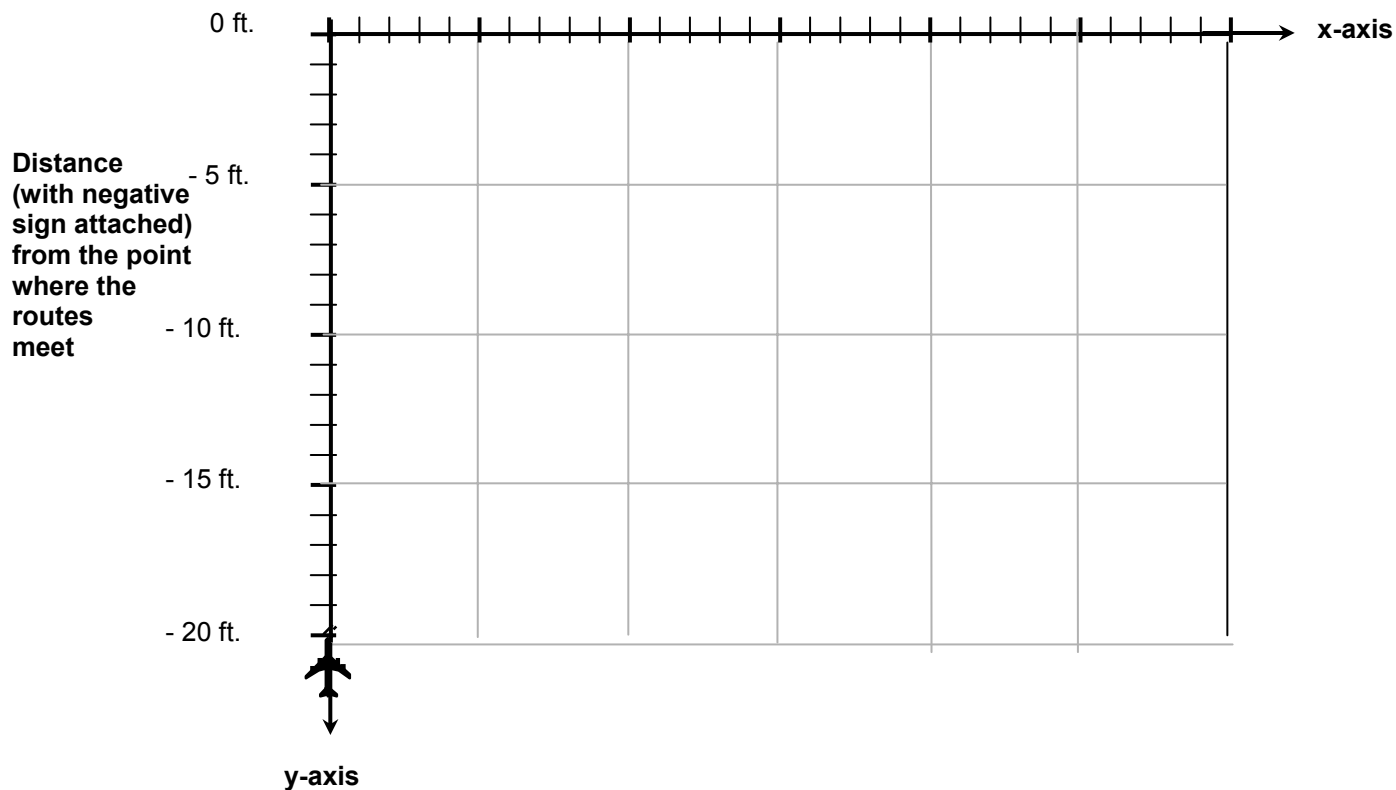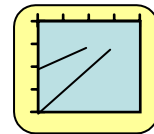
Use your graphs to answer the following questions.

1. How many seconds will it take each plane to travel 20 feet and arrive at the point

   where the two routes meet?

   WAL27 _____ seconds   NAL63 _____ seconds

2. Will the two planes meet at the point where

   the two routes come together?   _____

3. Why or why not? _____

_____

4. If you think the two planes will meet, what could you tell the air traffic

   controllers to do to avoid a collision? _____

5. Write the number that is the slope of the solid line representing
   Flight WAL27.   _____

6. Write the number that is the slope of the dotted line representing
   Flight NAL63.   _____

7. What information does the slope of each line tell you about each plane?

_____

_____

After the Experiment

Now you will compare your prediction with the results of the experiment.

First, circle your role in the experiment:

Pilot of WAL27	NASA Scientist for WAL27	Lead Air Traffic Controller
Pilot of NAL63	NASA Scientist for NAL63	Other Air Traffic Controller

----

Take a look at your prediction.
Did you predict the planes would meet at the point where the
two routes come together?   Yes   No

Next look at the results of the experiment.
Did the planes meet at the point where the two routes
come together?   Yes   No

Does your prediction match the experiment?   Yes   No

If your answer to the last question is No, why do you think your prediction and the
experiment do not match? _____
_____

Take another look at your prediction.

When the first plane gets to the point where the routes meet, how many feet away did
you think the second plane would be?

Does your prediction match the experiment?

If your answer is No, why do you think your prediction and the experiment do not
match? _____
_____

Take another look at the problem.

The speed of Flight WAL27 was 1/2 foot per second.

The speed of Flight NAL63 was 1/2 foot per second.

Were the speeds the same or different?

Same speed   Different speeds

Flight WAL27 started 20 feet from the point where the routes come together.

Flight NAL63 started 20 feet from the point where the routes come together.

Were the distances from the finish point the same or different?

Same distance   Different distances

Now think about this general problem.

Two planes are traveling at the same speed on two different routes.

Each plane is the same distance from the point where the two routes come together.

Will the planes meet at the point where the routes come together? _____

If not, how many feet apart will the planes be when the first plane reaches the point where the routes come together?_____

Explain your answers. _____

_____

Posttest

In the picture below, two airplanes are flying on different routes.

The speed of Flight WAL27 is 1 foot/second.

Flight WAL27 is 30 feet from the point where the two routes come together.

The speed of Flight NAL63 is 1 foot/second.

Flight NAL63 is 30 feet from the point where the two routes come together.

30 feet   30 feet

WAL27   NAL63
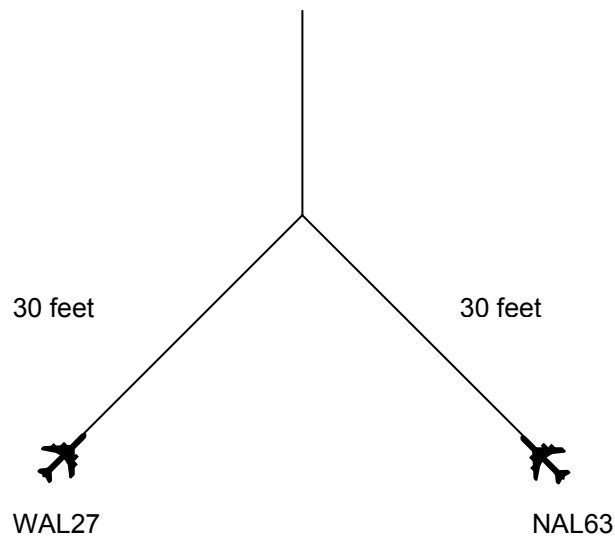
1. Do you think that the two planes will meet at the point
   where the two routes come together?   _____

   Why or why not? _____

   _____

2. If not, how far apart do you think the planes will be when the first plane
   reaches the point where the routes come together?   _____

Now consider this general problem.

Two planes are traveling at the same speed on two different routes.

Each plane is the same distance from the point where the two routes come together.

3. Will the planes meet at the point where the routes come together?_____

4. If not, how far apart will the planes be when the first plane reaches the point where the routes come together? _____

5. Explain your answers. _____
_____

6. If you think the two planes would meet, what could you tell the air traffic controllers to do to avoid a collision? _____

Curriculum Supplement 1

ANSWERS & EXPLANATIONS

A note on the organization of the Answers and Explanations:
In this Curriculum Supplement, most of the activities pose the same set of questions.
The answers to those questions are introduced in the first part of this answer document.
The remainder of the document is organized by activity and includes answers to
individual activity questions, graphs, applications of the distance-rate-time formula, and
discussions of the general problems posed in the analysis activity and the posttest.

The speed of Flight WAL27 is 1/2 foot/second, so the plane travels 1/2 foot in 1 second.
The speed of Flight NAL63 is 1/2 foot/second, so the plane travels 1/2 foot in 1 second.
Each plane is 20 feet from the point of intersection.

Since the planes are traveling at the same constant (fixed) speed and must travel the same
distance to the point of intersection, a conflict will occur at the intersection.

In particular:

- It will take 40 seconds for Flight WAL27 to travel 20 feet to the point where the
  routes come together.

- It will take 40 seconds for Flight NAL63 to travel 20 feet to the point where the
  routes come together.

- Since the times are the same for both airplanes, they will arrive at the same time and a
  conflict will occur.

To avoid a conflict, the air traffic controllers could change the speed or change the route
of one of the planes.

Activity 1.3A—Count Feet and Seconds:

Each plane travels 1 foot in 2 seconds. Count by 2s to complete the table.

Each plane will travel 20 feet in 40 seconds. (Students can also multiply 2 seconds per foot by 20 feet to obtain 40 seconds.) Since each plane takes 40 seconds to reach the intersection, they will arrive at the same time and a conflict will occur.

1. Fill in the given table to see how many seconds it will take each plane to travel 20 feet.

Flight WAL27		Flight NAL63	
How many feet?	How many seconds?	How many feet?	How many seconds?
1	2	1	2
2	4	2	4
3	6	3	6
4	8	4	8
5	10	5	10
6	12	6	12
7	14	7	14
8	16	8	16
9	18	9	18
10	20	10	20
11	22	11	22
12	24	12	24
13	26	13	26
14	28	14	28
15	30	15	30
16	32	16	32
17	34	17	34
18	36	18	36
19	38	19	38
20	40	20	40

Activity 1.3A (cont.)

After you fill in the table, answer the following questions:

2. How many seconds will it take each plane to travel 20 feet and arrive at the point where the two routes meet?

   WAL27 ____**40**____ seconds   NAL63 ____**40**____ seconds

3. Will the two planes meet at the point where the two routes come together?   __**Yes**__

4. Why or why not? **The planes each take 40 seconds to reach the intersection. So they arrive at the same time.**

5. You filled in the table to find the answer. Can you think of a faster way to find the answer? If so, describe the faster way. **Multiply 2 seconds per foot by   20 feet to obtain 40 seconds.**

6. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision?   **Change the speed or change the route of one of the planes.**

Activity 1.3B—Stacking Blocks:

Each plane travels 1 foot in 2 seconds. So in 10 seconds, each plane will go 5 feet. The following diagram shows the 5-foot blocks added together. To get the answer in seconds, add the 4 blocks of 10 seconds each to get 40 seconds. Since the times of arrival at the intersection are the same (40 seconds) for each plane, they will arrive at the same time and a conflict will occur.

Time

	Intersection		40 sec		Intersection		
0							0
	10 sec.				10 sec		
5			30 sec				5
	10 sec.				10 sec.		
10			20 sec				10
	10 sec.				10 sec.		
15			10 sec				15
	10 sec.				10 sec.		
20			Start				20

Distance from Intersection, feet

WAL27   NAL63

1. How many seconds will it take each plane to travel 20 feet and arrive at the point where the two routes meet?

     WAL27 ___**40**___ seconds   NAL63 ___**40**___ seconds

2. Will the two planes meet at the point where the two routes come together?   __**Yes**__

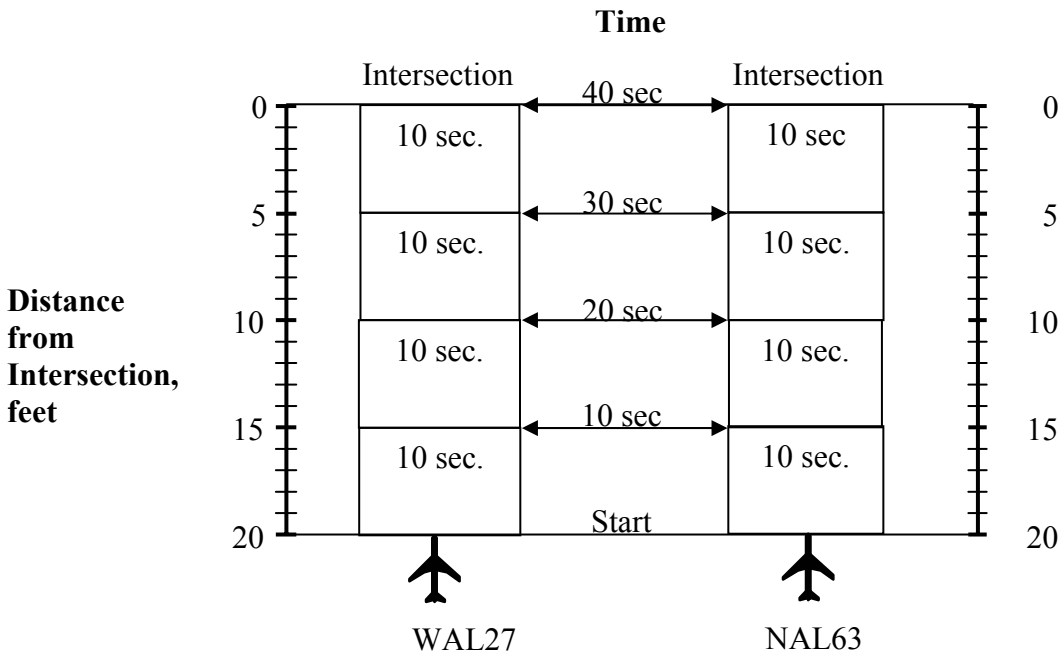3. Why or why not? **The planes each take 40 seconds to reach the intersection. So they arrive at the same time.**

4. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? **Change the speed or change the route of one of the planes.**

Activity 1.3C—Plot Points on Lines:

Each plane travels 1 foot in 2 seconds. So in 10 seconds, each plane will go 5 feet. The following diagram shows the position of each plane at 10-second intervals. Since the times of arrival at the intersection are identical (40 seconds) for each airplane, they will arrive at the same time and a conflict will occur.

1. How many seconds will it take each plane to travel 20 feet and arrive at the point where the two routes meet?

   WAL27 ____**40**____ seconds   NAL63 ____**40**____ seconds

2. Will the two planes meet at the point where
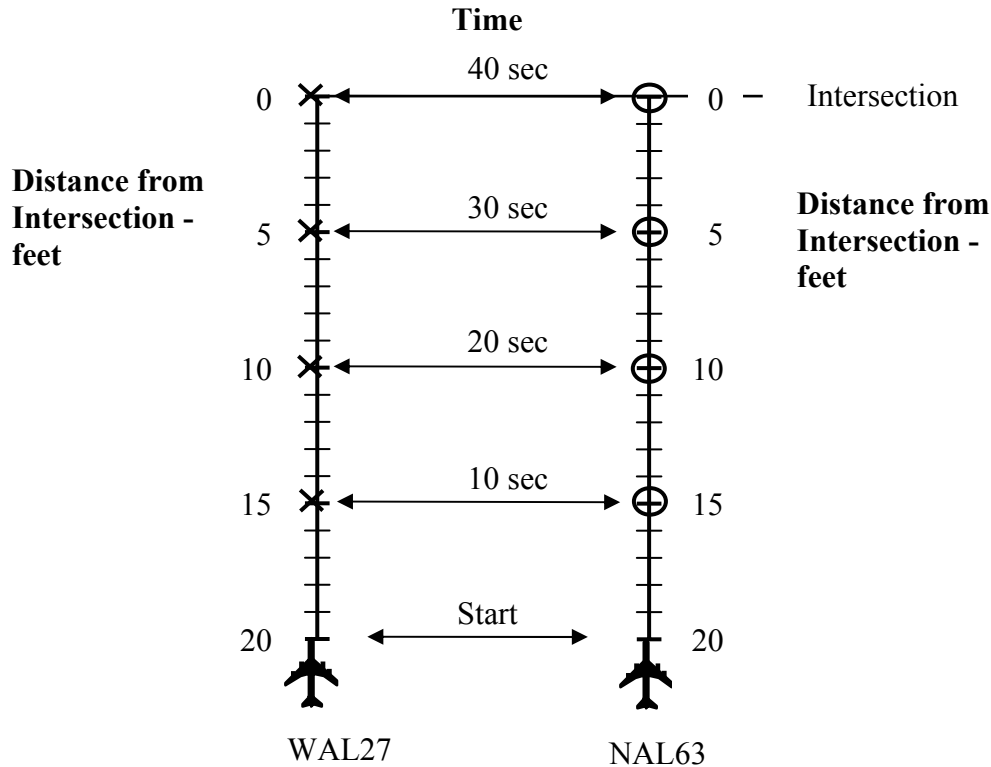   the two routes come together?   __**Yes**__

3. Why or why not? **The planes each take 40 seconds to reach the intersection. So they arrive at the same time.**

4. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? **Change the speed or change the route of one of the planes.**

Activity 1.3D—Plot Points on a Grid:

Each plane travels 1 foot in 2 seconds. So in 10 seconds, each plane will go 5 feet. The following graph shows the position of each plane at 10-second intervals. Since the times of arrival at the intersections are identical (40 seconds) for each plane, they will arrive at the same time and a conflict will occur.

1. How many seconds will it take each plane to travel 20 feet and arrive at the point where the two routes meet?

   WAL27 ____**40**____ seconds   NAL63 ____**40**____ seconds

2. Will the two planes meet at the point where
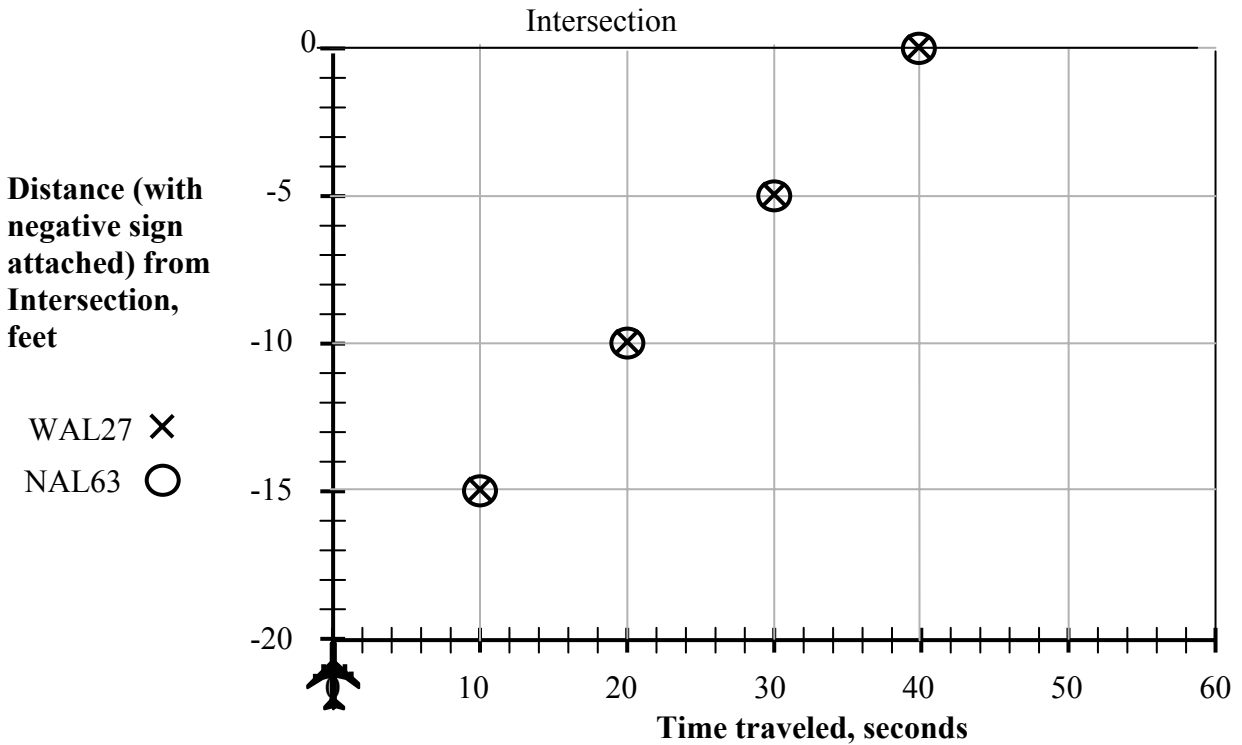   the two routes come together?   __**Yes**__

3. Why or why not?   **The planes each take 40 seconds to reach the intersection.**
   So they arrive at the same time.

4. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? **Change the speed or change the route of one of the planes.**

Activity 1.3E—Derive the Distance-Rate-Time Formula:

Each airplane starts 20 feet from the point where the two routes meet.

The speed of each airplane is 0.5 feet per second.

In 1 second, each plane travels 0.5 feet.

In 2 seconds, each plane travels 0.5 feet/second $\times$ 2 seconds $= 1.0$ foot.

In 3 seconds, each plane travels 0.5 feet/second $\times$ 3 seconds $= 1.5$ feet.

To find the distance each plane travels in 20 seconds, multiply 0.5 feet/second by 20. The result is 10.0 feet.

1. In 4 seconds, each plane travels __**0.5 ft./sec.**__ $\times$ __**4 sec.**__ = __**2.0**__ feet.
2. In 5 seconds, each plane travels __**0.5 ft./sec.**__ $\times$ __**5 sec.**__ = __**2.5**__ feet.
3. In 6 seconds, each plane travels __**0.5 ft./sec.**__ $\times$ __**6 sec.**__ = __**3.0**__ feet.

4. How could you use multiplication to find the distance each plane travels in
    14 seconds? __**Multiply 0.5 feet/second by 14 seconds.**__

5. Use the formula to find the distance traveled by each plane in 20 seconds.
    In 20 seconds, each plane travels __**10**__ feet.

Activity 1.3F—Use the Distance-Rate-Time Formula:

Calculate the time it takes each aircraft to travel 20 feet to the intersection.

The travel times are:

WAL27: t = 20 feet / 0.5 feet per second = 40 seconds

NAL63: t = 20 feet / 0.5 feet per second = 40 seconds

Since the times are the same, the airplanes will arrive at the same time and a conflict will occur.

Flight WAL27   t = $\dfrac{20\ feet}{0.5\ feet\ per\ second}$ = __40__ seconds

(Hint: Divide 20 by 0.5 .)

Flight NAL63   t = $\dfrac{\textbf{20 feet}}{\textbf{0.5 feet per second}}$ = __40__ seconds

1. How many seconds will it take each plane to travel 20 feet and arrive at the point where the two routes meet?

    WAL27 ___**40**___ seconds   NAL63 ___**40**___ seconds

2. Will the two planes meet at the point where
    the two routes come together?   __**Yes**__

3. Why or why not?   **The planes each take 40 seconds to reach the intersection.**
    So they arrive at the same time.

4. If you think the two planes will meet, what could you tell the air traffic controllers to do
    to avoid a collision? **Change the speed or change the route of one of the**
    planes.

Activity 1.3G—Graph Two Linear Equations:

Since the times of arrival at the intersections are identical (40 seconds) for each plane, they will arrive at the same time and a conflict will occur.

Each plane has the same corresponding equation, so the lines coincide.

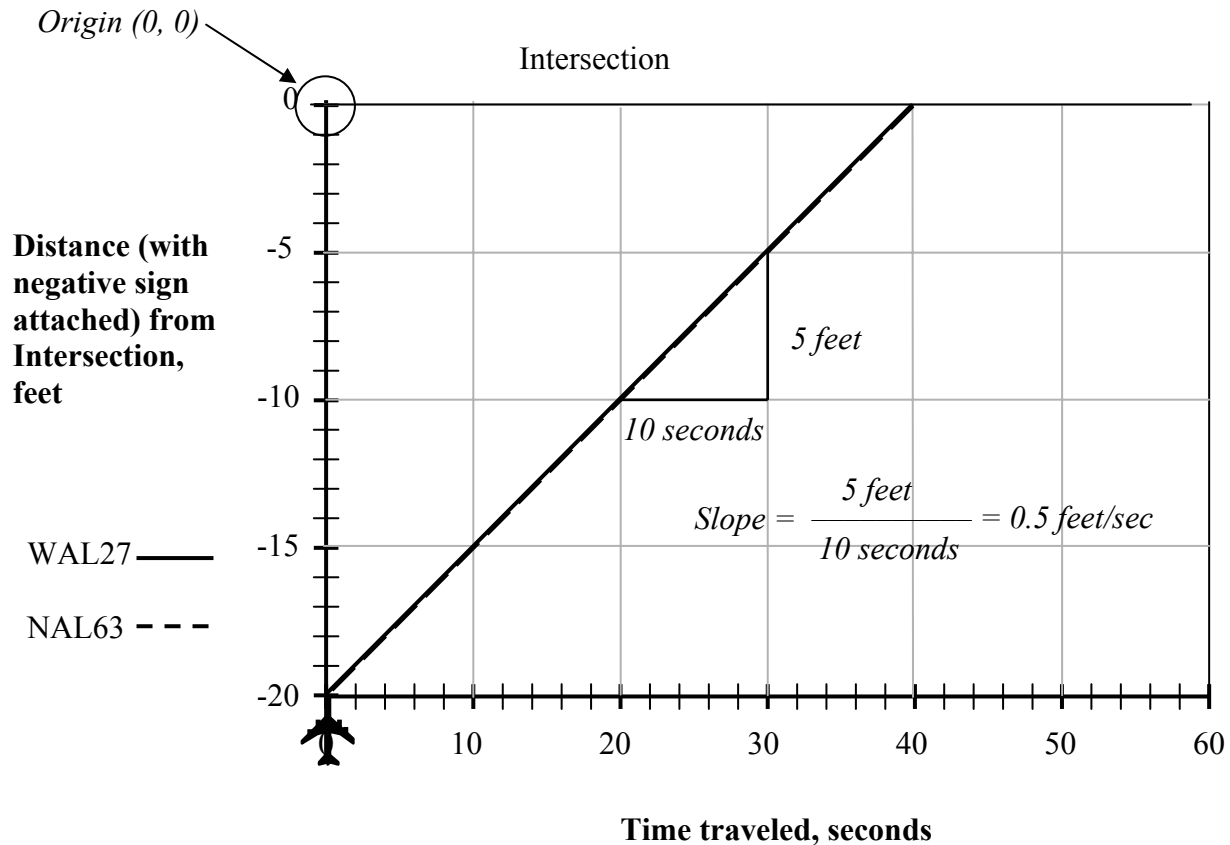The slope of the line is 0.5 feet/sec, the speed of each plane.
The vertical intercept of the line is –20. This corresponds to each plane's initial distance (20 feet) from the intersection.

WAL27   y = 0.5x – 20   NAL63   y = 0.5x – 20

x	y
0	-20
10	-15
20	-10
30	- 5

x	y
0	-20
10	-15
20	-10
30	- 5

Origin (0, 0)

Intersection

Distance (with negative sign attached) from Intersection, feet

5 feet

10 seconds

$$Slope = \frac{5\ feet}{10\ seconds} = 0.5\ feet/sec$$

WAL27 ———

NAL63 – – –

Time traveled, seconds

Activity 1.3G (cont.)

1. How many seconds will it take each plane to travel 20 feet and arrive at the point where the two routes meet?

   WAL27 ___**40**___ seconds   NAL63 ___**40**___ seconds

2. Will the two planes meet at the point where
   the two routes come together?   ___**Yes**___

3. Why or why not? **The planes each take 40 seconds to reach the intersection. So they arrive at the same time.**

4. If you think the two planes will meet, what could you tell the air traffic controllers to do to avoid a collision? **Change the speed or change the route of one of the planes.**

5. Write the number that is the slope of the solid line representing
   Flight WAL27.   **0.5 feet/second**

6. Write the number that is the slope of the dotted line representing
   Flight NAL63.   **0.5 feet/second**

7. What information does the slope of each line tell you about each plane?

   The slope of each line is 0.5 feet/second, the speed of the plane.

Activity 1.4—After the Experiment and Activity 1.5—Posttest:

The plane speeds are the same.
The planes are the same distance from the finish point.

General Problem:

Suppose two planes are traveling at the same speed on two different routes and each plane is the same distance from the point where the two routes come together.

Since the planes are traveling at the same speed and must travel the same distance to the point of intersection, the planes will arrive at the intersection at the same time. So the planes will meet at the point where the routes come together.